

Program VI. **GEOMETRY**

SOL Topic:

G.7

The student will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. Calculators will be used to solve problems and find decimal approximations for the solutions.

Activity 1: Use the Pythagorean Theorem to explore Algebraic relationships in Geometry.

The hypotenuse of a right triangle is 13 units. The difference of the lengths of the legs is 7 units. Find the length of each leg. Explore analytically, graphically, and numerically.

ANALYTICALLY :

Let:

- One leg = x
- Second leg = $(x - 7)$
- Hypotenuse = 13
- Equation to solve analytically is $x^2 + (x - 7)^2 = 13^2$
- Solution: first leg is $x = 12$, second leg $(x - 7)$ is 5.

GRAPHICALLY:

- $Y_1 = X^2 + (X - 7)^2$
- $Y_2 = 13^2$
- Use “2nd Calc”, “Intersect” to find solutions
- Solutions are $x = -5$, and $x = 12$, when $y = 169$ but a side of a triangle cannot be negative, therefore $x = 12$ is the solution.

NUMERICALLY:

- INVESTIGATE THE TABLES:
- When $x = -5$ and $x = 12$

Activity 2: Graph in Polar Mode, the Unit Circle with degree increments as the x and y coordinates correspond to the special right triangle values .

Mode:

- Polar
- Degree

Window:

- Zoom Decimal, divided by 3;

Graph and trace when:

- $r_1 = 1$

To observe the special properties of right triangles note the X and Y coordinates, when $\theta = 0, 30, 45, 60, 90$ degrees with an added bonus of 15 and 75 degrees at no charge.

Questions:

In which quadrant does a 120 degree angle fall?

What is the coterminal angle of -67.5 degrees?

In which quadrant does a -128.5 angle fall?

Can you find the sine, cosine, and tangent of 128.5 degrees?

What is the tangent of 45 degrees?

What is the sine of 30 degrees?

What is the cosine of 60 degrees?

When are the sine and cosine equal?

Activity 3: Graphing Simultaneously the Unit Circle and Sine Wave in Parametric Mode.

Mode:

- Parametric
- Simul
- Radian

- $X_{1T} = \cos(T) - 1$
- $Y_{1T} = \sin(T)$
- $X_{2T} = T$
- $Y_{2T} = \sin(T)$

Window:

T: [0, 6.29]

X: [-2.5, 6.29]

Y: [-3.1, 3.1]

Graph and Trace when $T = 1.57$, $T = 3.14$, $T = 4.71$, $T = 6.28$

View the circle and the sine curve using the “up” cursor when tracing.

Activity 4: Solving word problems in trigonometry using the home screen of the graphing calculator.

Problem:

A person is 6 feet tall and casts a shadow of 8 feet on the ground. At what angle is the sun above the horizon?

Is it morning, noon or evening? Explain.

Solution to finding the angle of the sun above the horizon:

- Degree Mode
- Home Screen
- $\tan^{-1}(6/8) = 36.86989765$ degrees

Change into degrees-minutes-seconds:

- 2nd ANGLE , #4: DMS
- ENTER
- Solution is 36 degrees 52 minutes, 11.632 seconds

G.13

The student will use formulas for surface area and volume of three dimensional objects to solve practical problems.

Calculators will be used to find decimal approximations for results.

Activity 5: Solving a practical problem in three dimensional geometry.

Find the amount of material required to construct a cone-shaped drinking cup for the Granby Elementary School water fountain. The slant height of the cone must be 5 inches, the radius of the cone is 4 inches. What is the height of the cone? What is the lateral area? What is the amount of water which the cone will hold?

Solution: From a 3-4-5 right triangle, if the radius = 4, then the height = 3.
Use the Formulas for finding the Lateral Area and the Volume of a cone :

$$L.A. = \pi r l$$

$$V = \frac{1}{3} \pi r^2 h$$

On the home screen:

$$L.A. = \pi * (4) (5)$$

$$L. A. = 62.83 \text{ inches}$$

$$V = \frac{1}{3} * \pi (4)^2 (3)$$

$$V = 50.27 \text{ cu.in.}$$

SOL Topic:

G.15

The student will draw a system of vectors and find the resultant graphically, write the components of a vector as a column matrix, and find the resultant by matrix addition; and solve practical problems using a system of vectors.

Activity 6: Given a system of vectors find the resultant with vector addition on the graph screen of the calculator.

Problem:

- Add the vectors $v_1(2,5)$, $v_2(4,3)$ to find the resultant $v_r(6,8)$ geometrically.

New Activity:

Window:

- X: $[-10, 84]_{10}$
- Y: $[-10, 52]_{10}$

Using the LINE command on the DRAW MENU:

- Draw a line segments (representing a vectors) on the home screen.
- Line (0, 0, 10, 25)
- Line (0, 0, 20, 15)
- Algebraically find the resultant.
- $(10, 25) + (20, 15) = (10+20, 25+15) \rightarrow (30, 40)$

- Graph the translation of the resultant using DRAW, “Line (
- Begin at the endpoint (10, 25), press Enter.
- Move the cursor on the graph screen right 20 units then move up 15 units, then press ENTER.
- Graph the resultant.
- Line (0, 0, 30, 40)

Activity 7: Writing the components of vectors in a column matrix then find the resultant by matrix addition within a practical problem applications.

The problem:

A tourist takes a taxi to go to the nearest pizza parlor in New York City. The taxi driver takes him 10 blocks east and 25 blocks north of his original location. He says, “This taxi driver is nuts” and gets out of the taxi and flags another. He says, “Please take me to your nearest pizza parlor.” The new taxi takes him 20 blocks west and 15 blocks north. When they arrived at the pizza parlor, how much did he pays both taxi drivers, and how many blocks from his point of origin had he traveled? (east-west, north-south)

Represent the vectors in a 2 by 1 column matrix.

Taxi 1 10
 25

Taxi 2 -20
 15

Location ?

Solution:

He had traveled 10 blocks west and 40 blocks north of his original location.
(Teacher can assume a charge per block in making an extension).